Name ANSWERS | Class

Total Marks

MATHS TEACHER HUB

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Circle theorems

(9 – 1) Topic booklet

Higher

These questions have been collated from previous years GCSE Mathematics papers.

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- If the question is a **1H** question you are not allowed to use a calculator.
- If the question is a **2H** or a **3H** question, you may use a calculator to help you answer.

Information

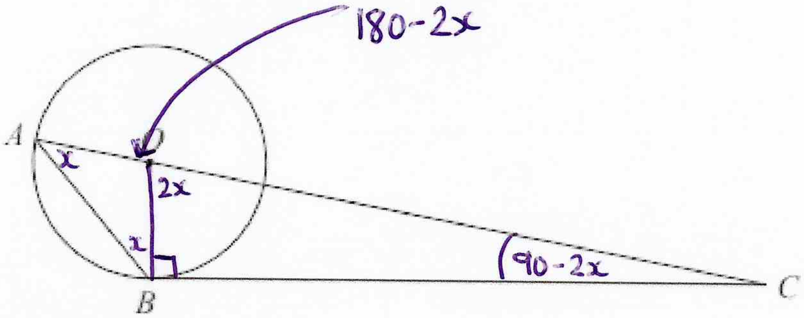
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions
Write your answers in the space provided.
You must write down all the stages in your working.

11



A and B are points on a circle, centre O.

BC is a tangent to the circle.
AOC is a straight line.
Angle $ABO = x°$.

Find the size of angle ACB, in terms of x.
Give your answer in its simplest form.
Give reasons for each stage of your working.

OBC = 90° tangent meets a radius at 90°

BAO = ABO = x Base angles of an isosceles triangle are equal

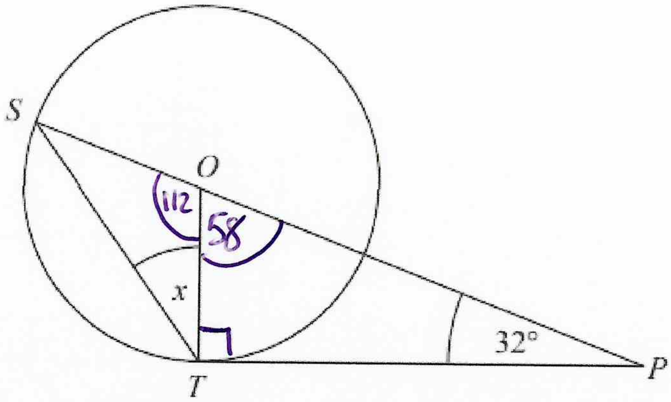
AOB = 180-2x Angles in a triangle add to 180°

BOC = 2x Angles on a straight line add to 180°

ACB = 90-2x Angles in a triangle add to 180°

(Total for Question 11 is 5 marks)

11



S and T are points on the circumference of a circle, centre O.
PT is a tangent to the circle.
SOP is a straight line.
Angle $OPT = 32°$

Work out the size of the angle marked x.
You must give a reason for each stage of your working.

OTP = 90° tangent meets a radius at 90°

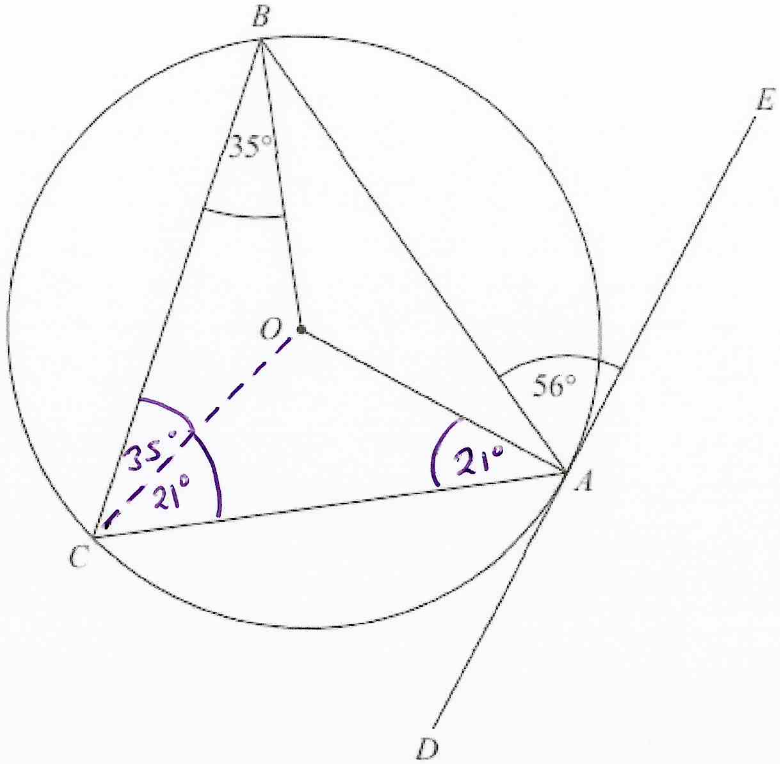
TOP = 58° Angles in a triangle add to 180°

SOT = 122° Angles on a straight line add to 180°

x = 29° base angles of an isosceles triangle are equal.

(Total for Question 11 is 4 marks)

12



A, B and C are points on the circumference of a circle, centre O.
DAE is the tangent to the circle at A.

Angle $BAE = 56°$
Angle $CBO = 35°$

Work out the size of angle CAO.
You must show all your working.

BCA = 56° Angles that lie between a tangent and a chord, are equal to the angle subtended by the same chord in the alternate segment.

OCB = 35° base angles of an isosceles triangle are equal

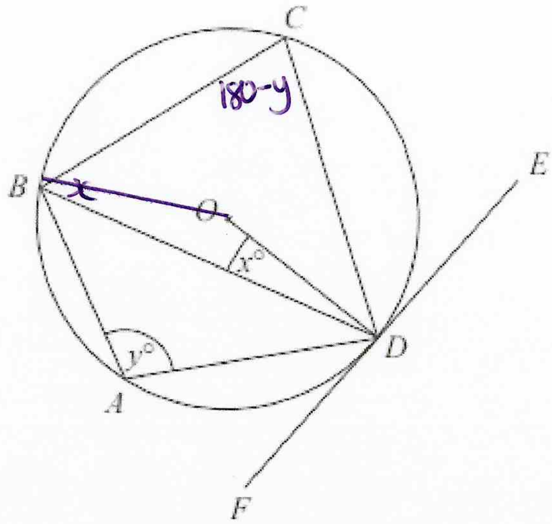
OCA = 21° because 56° − 35°

CAO = 21° base angles of an isosceles triangle are equal

21 °

(Total for Question 12 is 3 marks)

13



A, B, C and D are points on the circumference of a circle, centre O.
FDE is a tangent to the circle.

(a) Show that $y - x = 90$
You must give a reason for each stage of your working.

OBD = x   Base angles of an isosceles triangle are equal

BCD = 180y   opposite angles of a cyclic quadrilateral add to 180°

BOD = 360 - 2y   Angles at the centre, are double the size of angles at the circumference.

$x + x + 360 - 2y = 180$
$2x + 180 - 2y = 0$
$x + 90 - y = 0$

$x - y = -90$
$y - x = 90$

(3)

Dylan was asked to give some possible values for x and y.

He said,
"y could be 200 and x could be 110, because $200 - 110 = 90$"

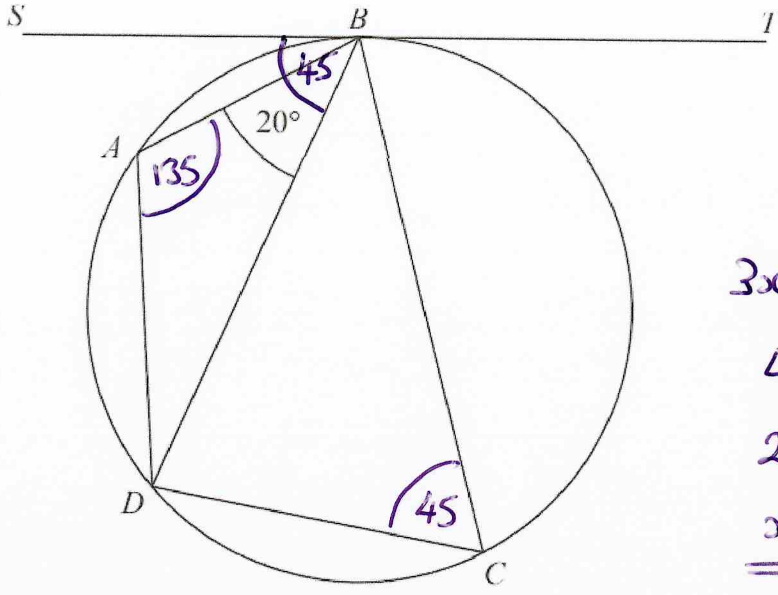
(b) Is Dylan correct?
You must give a reason for your answer.

No y must be less than 180° as it is an angle inside a triangle

(1)

(Total for Question 13 is 4 marks)

14



$3x + x = 180$

$4x = 180$

$2x = 90$

$\underline{\underline{x = 45°}}$

A, B, C and D are four points on a circle.
SBT is a tangent to the circle.
Angle $ABD = 20°$

the size of angle BAD : the size of angle $BCD = 3:1$

Find the size of angle SBA.
Give a reason for each stage of your working.

BAD and DCB = 180 Opposite angles of a cyclic quadrilateral add to 180°

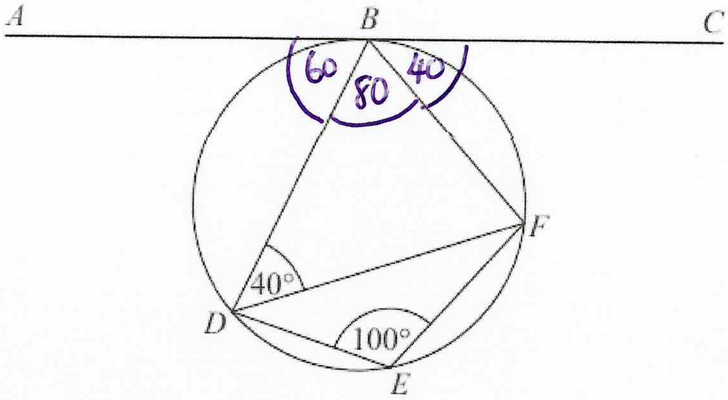
SBD = 45° The angle that lies between a tangent and a chord is equal to the angle subtended by the same chord in the alternate segment.

SBA = 25° 45-20=25

25 .... °

(Total for Question 14 is 4 marks)

14



Points B, D, E and F lie on a circle.
ABC is the tangent to the circle at B.

Find the size of angle ABD.
You must give a reason for each stage of your working.

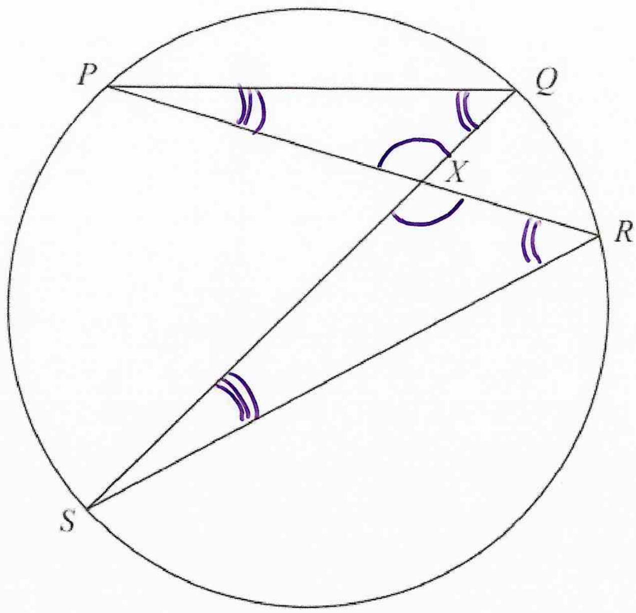
DBF = 80° Opposite angles of a cyclic quadrilateral add to 180°

CBF = 40° The angle that lies between a tangent and a chord, is equal to the angle subtended by the same chord in the alternate segment.

ABD = 60° Angles on a straight line add to 180°

(Total for Question 14 is 4 marks)

15 P, Q, R and S are four points on a circle.



PXR and SXQ are straight lines.

Prove that triangle PQX and triangle SRX are similar.

PXQ = SXR   vertically opposite angles are equal.

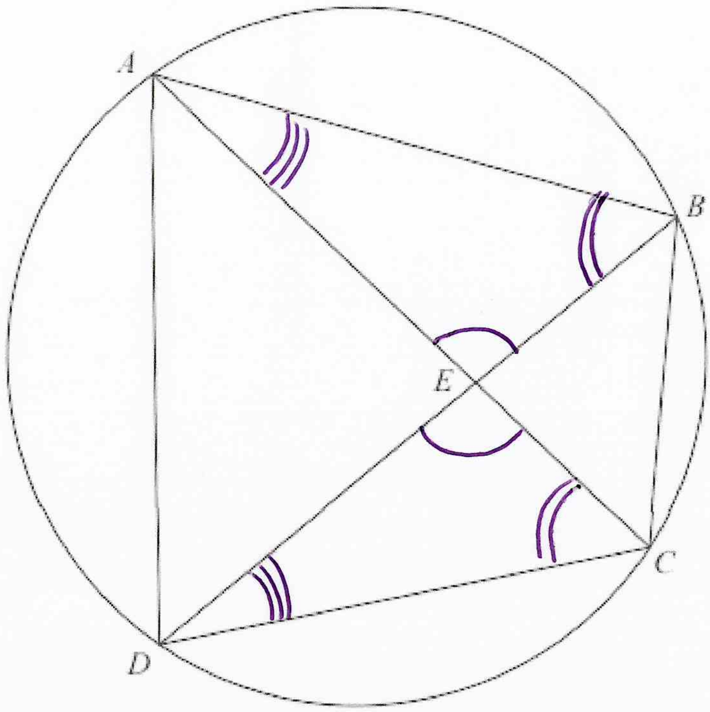
PQS = PRS   Angles in the same segment are equal.

RSQ = RPQ   Angles in the same segment are equal.

Both triangles are similar as they have three identical angles (AAA)

(Total for Question 15 is 3 marks)

15 A, B, C and D are four points on the circumference of a circle.



AEC and BED are straight lines.

Prove that triangle ABE and triangle DCE are similar.
You must give reasons for each stage of your working.

AEB = DEC   Vertically opposite angles are equal

ABE = ECD   Angles from the same chord are equal

BAE = BDC   Angles from the same chord are equal

Both triangles are similar as the have
three identical angles (AAA)

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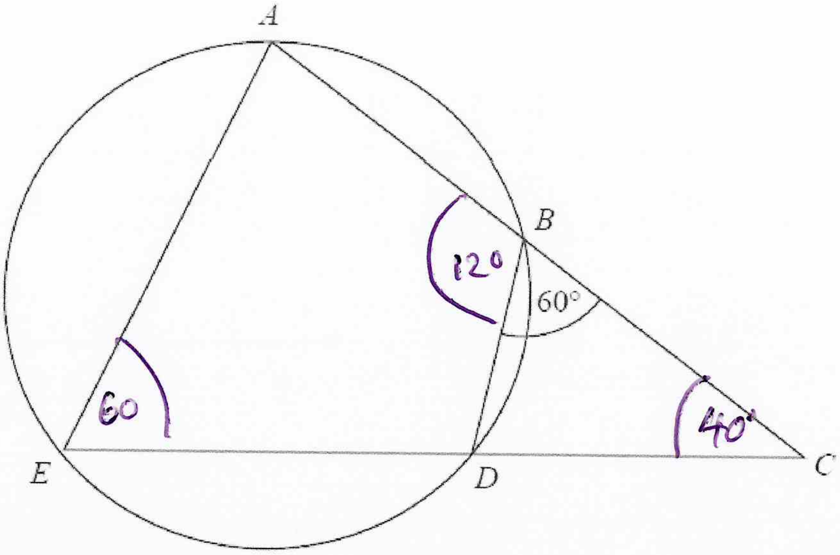
(Total for Question 15 is 3 marks)

16

ABDE is a cyclic quadrilateral.
ABC and *EDC* are straight lines.
Angle $DBC = 60°$

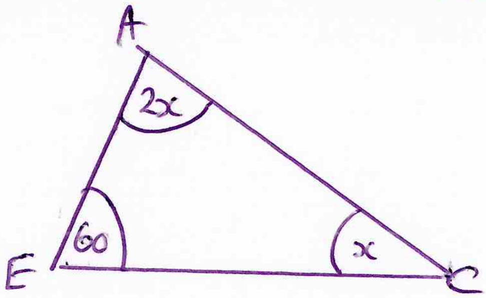
Given that

size of angle EAB : size of angle BCD = 2 : 1

work out the size of angle *BCD*.
You must show all your working.

ABD = 120° Angles on a straight line add to 180°

AED = 60° Opposite angles of a cyclic quadrilatera add to 180°



$60 + 2x + x = 180$

$60 + 3x = 180$

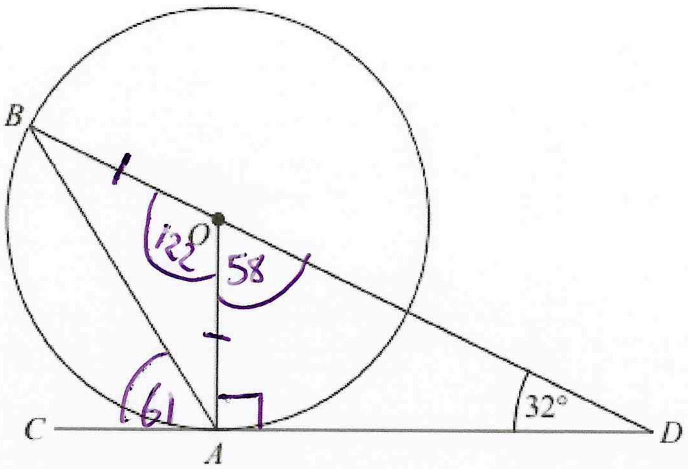
$3x = 120$

$x = 40$

40 °

(Total for Question 16 is 4 marks)

17



A and B are points on a circle with centre O.
CAD is the tangent to the circle at A.
BOD is a straight line.

Angle $ODA = 32°$

Work out the size of angle CAB.
You must show all your working.

OAD = 90° tangent meets a radius at 90°

DOA = 58° Angles in a triangle add to 180°

BOA = 122° Angles on a straight line add to 180°

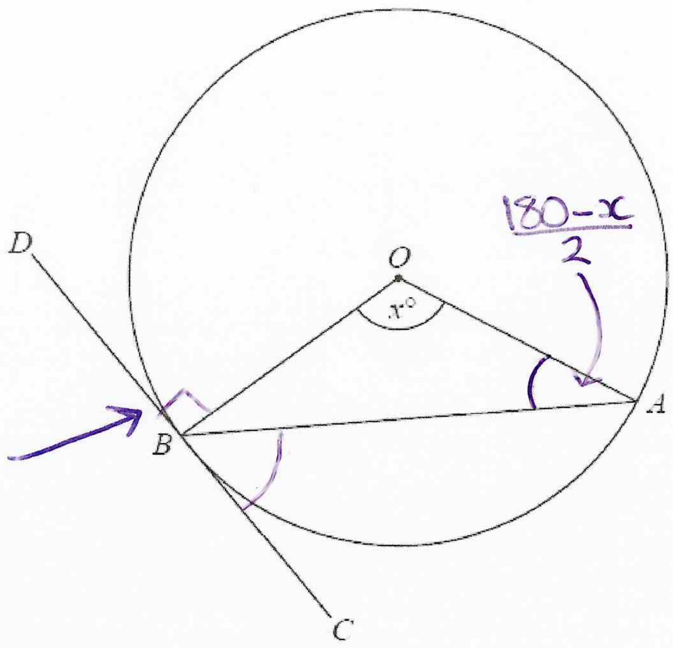
OAB and OBA = 29° base angles of an isosceles triangle are equal.

CAB = 61° 90 - 29 = 61

61 °

(Total for Question 17 is 3 marks)

Tangent meets a radius at 90°



Base angles of an isosceles triangle are equal

A and B are points on a circle, centre O.
DBC is the tangent to the circle at B.
Angle $AOB = x°$

Show that angle $ABC = \frac{1}{2}x°$

You must give a reason for each stage of your working.

$$ABC = 180 - 90 - \left(\frac{180 - x}{2}\right)$$

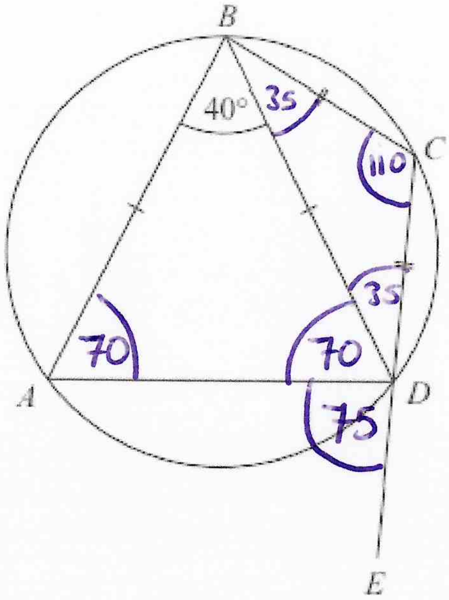
$$= 90 - 90 + \frac{x}{2}$$

$$= \frac{x}{2}$$

$$= \frac{1}{2}x$$

(Total for Question 18 is 3 marks)

18 The points A, B, C and D lie on a circle.
CDE is a straight line.



$BA = BD$
$CB = CD$
Angle $ABD = 40°$

Work out the size of angle ADE.
You must give a reason for each stage of your working.

BAD = BDA = 70° base angles of an isosceles triangle are equal

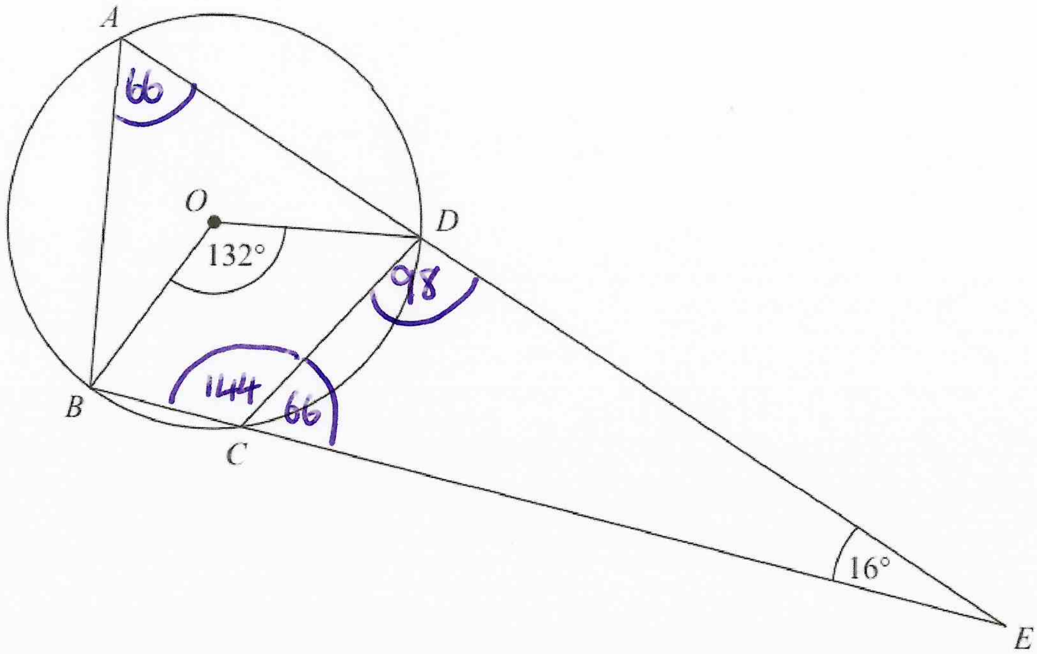
BCD = 110° opposite angles of a cyclic quadrilateral add to 180°

CDB = CBD = 35° base angles of an isosceles triangle are equal.

ADE = 75° Angles on a straight line add to 180°

(Total for Question 18 is 5 marks)

20 A, B, C and D are points on the circumference of a circle, centre O.
ADE and BCE are straight lines.



Work out the size of angle CDE.
Give a reason for each stage of your working.

BAD = 66° Angles at the centre are double the size of an angle at the circumference.

BCD = 114° opposite angles of a cyclic quadrilateral add to 180°

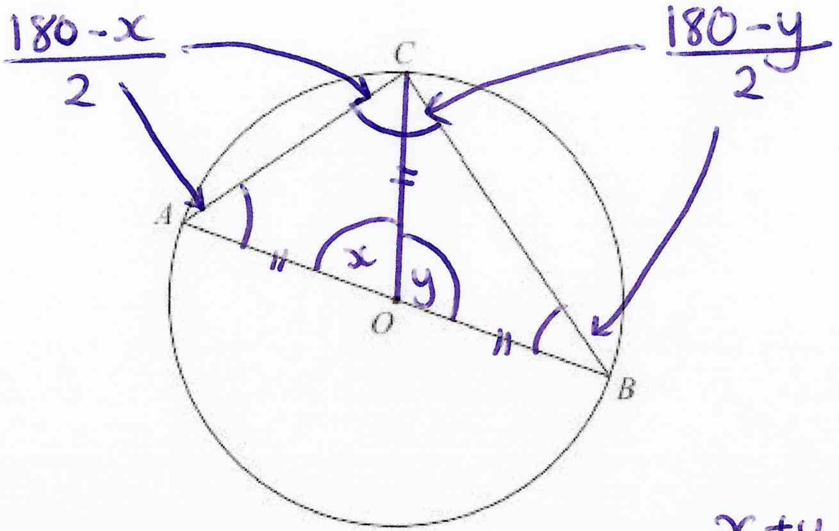
DCE = 66° Angles on a straight line add to 180°

CDE = 98° Angles in a triangle add to 180°

98 °

(Total for Question 20 is 4 marks)

20



$x + y = 180$

$\frac{1}{2}x + \frac{1}{2}y = 90$

A, *B* and *C* are points on the circumference of a circle, centre *O*.
AOB is a diameter of the circle.

Prove that angle *ACB* is 90°
You must **not** use any circle theorems in your proof.

$OAC = OCA = \frac{180 - x}{2} = 90 - \frac{1}{2}x$

$OBC = OCB = \frac{180 - y}{2} = 90 - \frac{1}{2}y$

$ACB = 90 - \frac{1}{2}x + 90 - \frac{1}{2}y$

$= 180 - \frac{1}{2}x - \frac{1}{2}y$

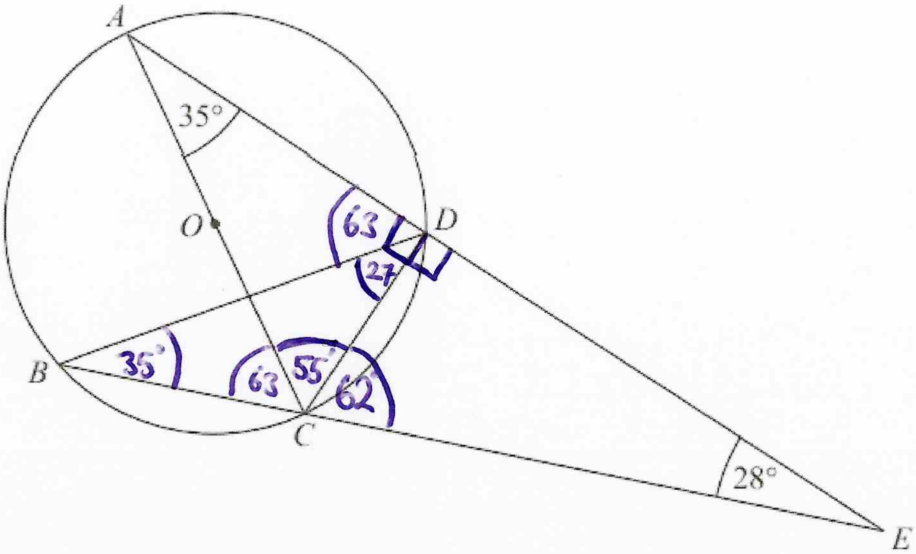
$= 180 - \left(\frac{1}{2}x + \frac{1}{2}y\right)$

$= 180 - 90$

(Total for Question 20 is 4 marks)

$= 90$

21



A, B, C and D are points on the circumference of a circle, centre O.
AC is a diameter of the circle.

ADE and BCE are straight lines.

Work out the size of angle BDC.
Write down any circle theorems that you use.

DBC = 35° Angles in the same segment are equal

ADC = 90° Angle in a semi circle is a right angle.

DCE = 62° Angles in a triangle add to 180°

ACD = 55° Angles in a triangle add to 180°

BCA = 63° Angles on a straight line add to 180°

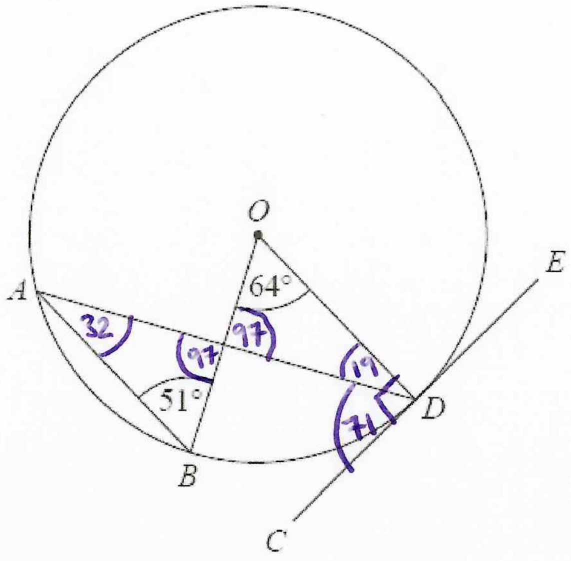
ADB = 63° Angles in the same segment

BDC = 27° 90 − 63 = 27

27 °

(Total for Question 21 is 4 marks)

21 A, B and D are points on a circle with centre O.
CDE is the tangent to the circle at D.



Work out the size of angle ADC.
Write down any circle theorems you use.

BAD = 32° Angles at the centre are double the size of angle at the circumference.

Angle in the middle = 97° because angles in a triangle add to 180°

ODA = 19° Angles in a triangle add to 180°

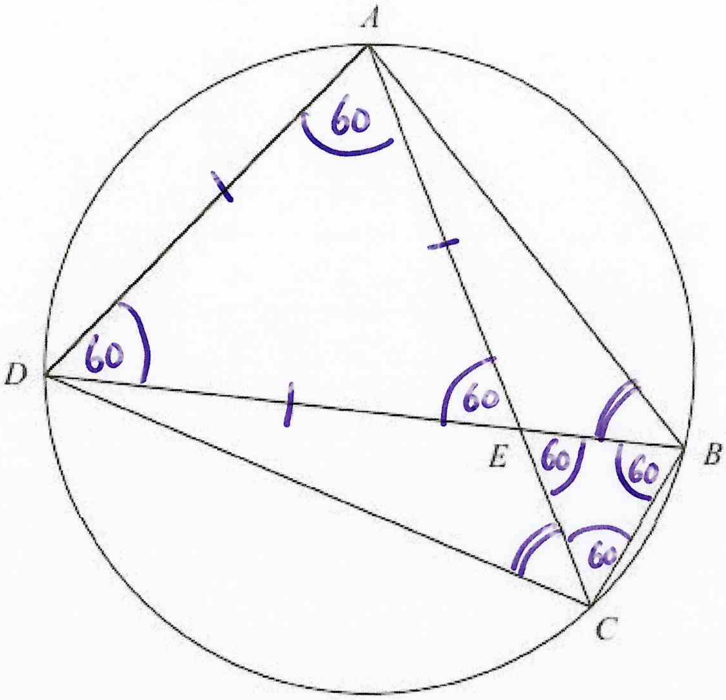
ADC = 71° When a tangent meets a radius you get 90°

90 - 19 = 71

71 °

(Total for Question 21 is 4 marks)

22 A, B, C and D are four points on a circle.



AEC and DEB are straight lines.

Triangle AED is an equilateral triangle.

Prove that triangle ABC is congruent to triangle DCB.

BC is common to both triangles

DBC = 60° Angles that subtend the same chord are equal

ACB = ADB " " " "

ACD = ABD " " " "

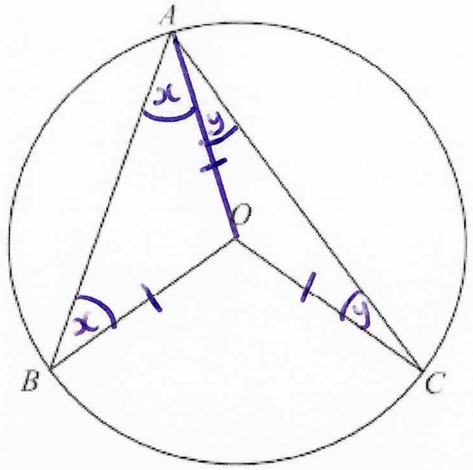
Triangle ABC is congruent to DCB

DBC = ACB = 60°
ABC = BCD

Angle side Angle.

(Total for Question 22 is 4 marks)

24 A, B and C are points on the circumference of a circle centre O.



Prove that angle BOC is twice the size of angle BAC.

$AOB = 180 - 2x$ $\quad\quad$ $AOC = 180 - 2y$

Angles in a triangle add to 180°

$BOC = 360 - (180 - 2x) - (180 - 2y)$

$= 360 - 180 + 2x - 180 + 2y$

$= 2x + 2y$

Angles around a point add to 360°

$BAC = x + y$

$BOC = 2x + 2y$

$= 2(x + y)$

(Total for Question 24 is 4 marks)